BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B69 09039

SUBJECT: Use of the Landing Point Designator to Land the Lunar Module in a Circular Target Area - Case 310

DATE: September 16, 1969

FROM: K. P. Klaasen

ABSTRACT

The probability of landing the lunar module (LM) at a point of scientific interest can be increased through efficient use of the landing point designator (LPD). The probability of making a successful redesignation to the desired target is maximized by properly biasing the initial aim point. Increasing the acceptable target from a point to a circular area allows a redesignation to the target area to be made with the same probability of success using a smaller redesignation delta-V budget. Circular target areas can be considered whenever the astronauts' radius of mobility on the lunar surface is great enough to allow them to reach the points of scientific interest from any point within the target area and whenever acceptably smooth landing points are distributed throughout the area. In such cases, a substantial savings in redesignation delta-V budget can be realized over that required to land at a point target.

The probability of success and the location of the optimum initial aim point depend upon the relationship between the redesignation capability, LM landing accuracy, and the target area radius in any given case. Assuming the Apollo 11 LM descent trajectory and automatic landing error ellipse, for a single, exact redesignation made at an altitude of 5000 ft. a point landing can be achieved with a 50% probability of success with a redesignation delta-V budget of about 460 fps. However, a landing within a circle of radius 5400 ft. can be achieved with 50% probability of success with a zero delta-V budget. The relationship between landing target radius and the required redesignation capability is approximately linear.

(NASA-CR-106041) USE OF THE LANDING POINT DESIGNATOR TO LAND THE LUNAR MODULE IN A CIRCULAR TARGET AREA (Bellcomm, Inc.) 22 p

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MEMORANDUM FOR FILE

INTRODUCTION

The planning of future lunar exploration missions has increased interest in the use of the landing point designator (LPD) to land the lunar module (LM) at a point of scientific interest. Prior studies have considered the use of the LPD to land the LM at a given point target using a single redesignation.[1] A logical extension of this problem is to the case of a circular target area. By allowing the acceptable landing point to be anywhere within an area of radius Δ about the true target point, the same degree of probable success in reaching the target can be achieved with a smaller redesignation delta-V budget. Acceptance of less accuracy in landing thereby brings about a savings in the overall propellant requirement. The degree of reduced landing accuracy which can be accepted is dependent upon the points to be explored in any particular mission and on the terrain in the vicinity of the target. If the mission requires exploration of a single specific point, the acceptable target area could be a circle centered at the point with radius equal to the astronauts' radius of mobility on the moon walking and/or using some type of mobility aid. If the mission requires the exploration of several points distributed throughout a limited area or a point near which landing is impossible due to rough terrain, the acceptable target area could be any circle such that all the points of scientific interest can be reached from any point within the circle given the astronauts' radius of mobility. In missions in which a mobility aid is included in the LM payload, the required landing accuracy of the LM is reduced. The resulting redesignation delta-V savings helps to compensate for the increase in LM payload weight due to the inclusion of the mobility aid.

REDESIGNATION ASSUMPTIONS

The problem is to determine the method of using the LPD such that the probability of landing the LM within a specific circular target area is maximized for various redesignation delta-V budgets.

It is assumed that:

(1) Due to the target visibility constraint of the LM window, only left and/or downrange landing point redesignations are permitted.

(2) Redesignation capability in distance is determined by the redesignation delta-V budget and the altitude at which the redesignation is made as illustrated in Figures 1a and 1b.[2]

(3) Dispersions are normally distributed about the initial aim point.

(4) Only a single redesignation is made.

(5) The redesignation is effected instantaneously.

BIASED TARGETS FOR ONE-DIMENSIONAL REDESIGNATIONS

The determination of the probability of landing within a circular area involves consideration of a two-dimensional normal distribution. The method of solution can be illustrated by means of the special one-dimensional case. First the case of the point target is considered as presented in Reference 1. In Figure 2a, the normal distribution function, ϕ, with standard deviation σ_Y, is plotted along the Y axis. Let T denote the target point and ρ the total redesignation capability (left hand only) of the LPD. O is the point toward which the LM is initially aimed. If the eventual landing point of the LM as designated by the LPD were anywhere within the interval T to T + ρ, it would be possible to redesignate the LM to T. However, if the designated landing point were outside of this interval, a redesignation to T would not be possible. The probability of landing within (T, T + ρ) is given by the area under the normal distribution curve over the interval, i.e.,

$$P = \frac{1}{\sigma_Y \sqrt{2\pi}} \int_T^{T + \rho} \exp \left[- \frac{(Y - \mu_Y)^2}{2\sigma_Y^2} \right] dy$$

where μ_Y is the value of the mean, and σ_Y is the standard deviation from the mean. Thus, the probability of landing within (T, T + ρ) is maximized by placing the initial aim point O at the center of the interval as shown in Figure 2b.

The probability of landing at T using a single redesignation is maximized, then, if the initial aim point is biased to the right of the actual desired landing point by the distance $\rho/2$.

Should the acceptable target now be allowed to include all points within a distance Δ of T, as depicted in Figure 3a, any designated landing point within the interval $(T - \Delta, T + \Delta + \rho)$ allows for a redesignation to the target to be made. The initial aim point is still biased to the center of the interval, as shown in Figure 3b, but the probability of making a successful redesignation to the target is increased since the normal curve in this case is integrated over an interval of length $\rho + 2\Delta$.

BIASED TARGETS FOR TWO-DIMENSIONAL REDESIGNATIONS

The generalization to the case of a two-dimensional target is presented in Figure 4. The Y-axis is crossrange and the X-axis is downrange. The target point or area is T, and O is the initial aim point about which the two-dimensional normal distribution is centered. ρ_x and ρ_y are the downrange and crossrange redesignation capabilities at the time of redesignation for a specified redesignation delta-V budget. Δ is the target area radius. The two-dimensional redesignation capability of the LPD is assumed to be bounded by a quarter ellipse with semi-axes of ρ_y and ρ_x. Data which specify exactly the current redesignation capability footprint are lacking; however, results based upon an earlier LM descent trajectory show that an elliptical boundary is a reasonable approximation.[3] Any designated landing point within the area R, which is bounded by the quarter ellipse of semi-axes $\rho_y + \Delta$ and $\rho_x + \Delta$ as shown in Figure 4, permits a redesignation to the target within the established constraints. The probability of landing in R is given by

$$P = \frac{1}{2\pi\,\sigma_x\,\sigma_y} \iint_R \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - \frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] dy\,dx$$

where μ_x and μ_y are the coordinates of the aim point O, and σ_x and σ_y are the downrange and crossrange standard deviations of the LM automatic landing error ellipse. The probability is maximized by choosing the proper coordinates μ_x and μ_y.

RESULTING OPTIMUM AIM POINTS AND PROBABILITIES

The optimum initial aim point and the probability of making a successful redesignation to the target were calculated for ratios of σ_y to σ_x of 1:5, 1:2, 1:1.5, 1:1, 1.5:1, and 2:1 and several values of the landing target radius and redesignation capability. In Figures 5a, b, c, d, e, and f, the radius of the landing target area is plotted vs. the crossrange redesignation capability for fixed values of the probability of making a successful redesignation to the target. All distance measurements are expressed in units of σ_y to allow for the greatest degree of generality. Thus, the results will remain applicable in spite of changes in the LM automatic landing error ellipse and/or the redesignation delta-V budget. For redesignation delta-V budgets of less than 100 feet per second, crossrange redesignation capability is approximately equal to twice the downrange capability ($\rho_y = 2\rho_x$).[4] However, for larger delta-V budgets in the area of 400 fps, this ratio is approximately reversed as shown in Figures 1a and 1b. Results for both ratios of ρ_x to ρ_y are plotted to show the range over which the results may vary as the redesignation delta-V budget changes. In each case, the optimum initial aim point was chosen. Figures 6a and 6b show how the coordinates μ_x and μ_y of the optimum aim point vary with redesignation capability for $\rho_y = 2\rho_x$. The greater the redesignation capability, the further the aim point O must be biased to the right and uprange of the true target in order to allow for the greatest probability of successfully redesignating to the target. In all cases the redesignation to the target is assumed to be made without error. In effect this means that the maneuver must be made at an altitude close enough to the lunar surface that the error involved in LPD operation is less than the translational capability still available during the manually controlled hover period immediately before final touchdown.

A better feeling for the actual numbers involved can be achieved through examination of this problem for redesignations from the Apollo 11 LM descent trajectory. For that mission the LM automatic landing error ellipse was given by σ_x = 6481 ft. and σ_y = 2836 ft.[5] A single redesignation is assumed to be made at an altitude of 5000 ft. Redesignation capability is as presented in Figures 1a and 1b. The results are shown in Figure 7. Since the slope of the constant probability curves is small, a very significant redesignation delta-V savings can be realized for only a relatively small increase in the landing target radius. For instance, a landing at a point target can be achieved with a 50% probability of success for a redesignation delta-V budget of about 460 fps. However, the same probability of successfully reaching the target can be maintained with zero delta-V budget if the target is allowed to be a circle of radius 5400 ft. This delta-V savings is equivalent to about 1400 lbs of LM payload.

CONCLUSIONS

The probability of making a successful redesignation to a given target area using the LPD to make left and/or downrange redesignations is maximized by properly biasing the initial aim point to the right and uprange of the true target. For all fully automatic landing ellipses considered, the relationship between landing target radius and the redesignation capability required to land within this target area with a given probability of success is approximately linear. An increase in landing target radius of one unit of distance allows a corresponding decrease in redesignation capability of about three units of distance with $\rho_y = 2\rho_y$ and about one and a half units of distance with $\rho_x = 2\rho_y$ for an equivalent probability of success. For the Apollo 11 LM descent trajectory and automatic landing error ellipse, and assuming an exact redesignation at 5000 ft., an increase of 1000 ft. in landing target radius allows a decrease in the redesignation delta-V requirement of about 75 fps. Thus, an increase in the acceptable landing site area could result in significant savings in the redesignation delta-V budget without decreasing the probability of landing within the target area.

K. P. Klaasen

2013-KPK-srb

K. P. Klaasen

Attachments:
References
Figures 1 through 7

References

1. Suddath, J. H. and R. L. Saldana, "Use of the Landing Point Designator to Land the Lunar Module to a Given Target", MSC Internal Note 67-EG-32, November 14, 1967.

2. LaPiana, F., "Preliminary Results for LPD Pulse Costs and Effects", Bellcomm Memorandum for File in preparation, July 1969.

3. Montgomery, J. D. and R. H. Lefler, "LM Landing Point Designator Procedures and Capability", MSC Internal Note 67-EG-24, August 1, 1967.

4. Hoekstra, T. B., "Presentations on LM descent at 8:30 APO Staff Meetings of June 10, 12 and 16, 1969 - Case 310", Bellcomm Memorandum for File, June 23, 1969.

5. "Apollo Mission G Mission Planning Briefing for the Crew", MSC Internal Note 69-FM-77, March 28, 1969.

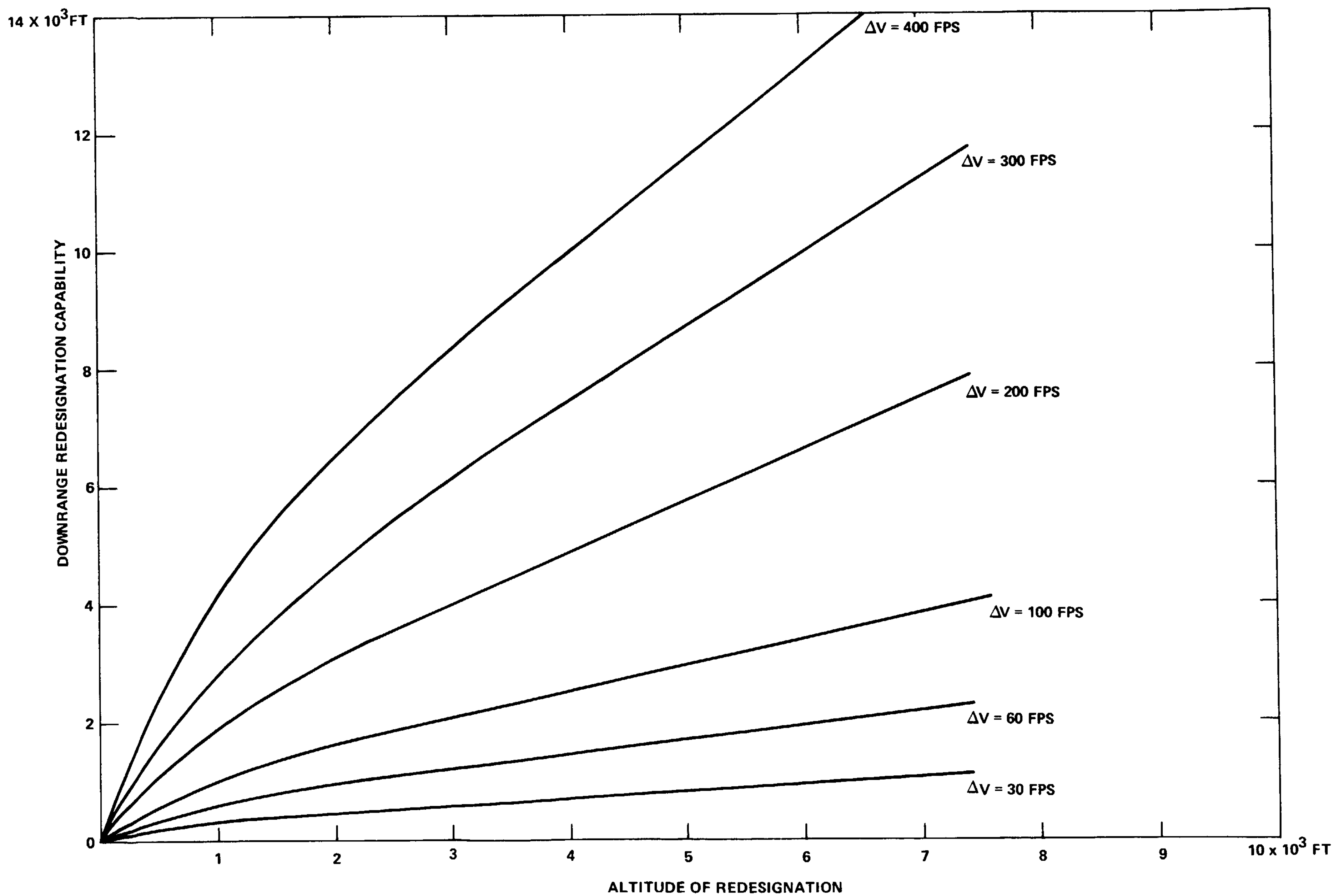

FIGURE 1A - DOWNRANGE REDESIGNATION CAPABILITY

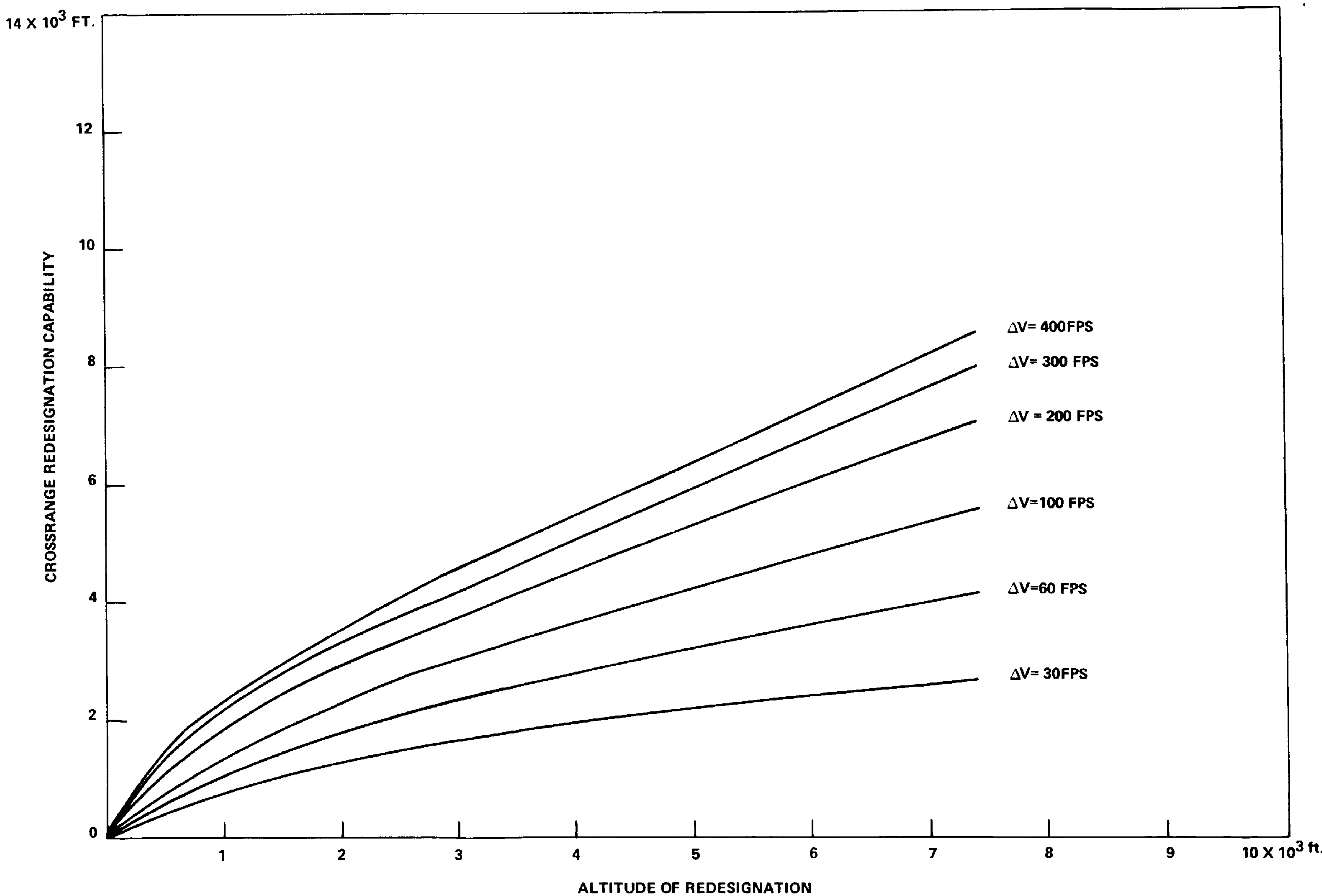

FIGURE 1B- CROSSRANGE REDESIGNATION CAPABILITY

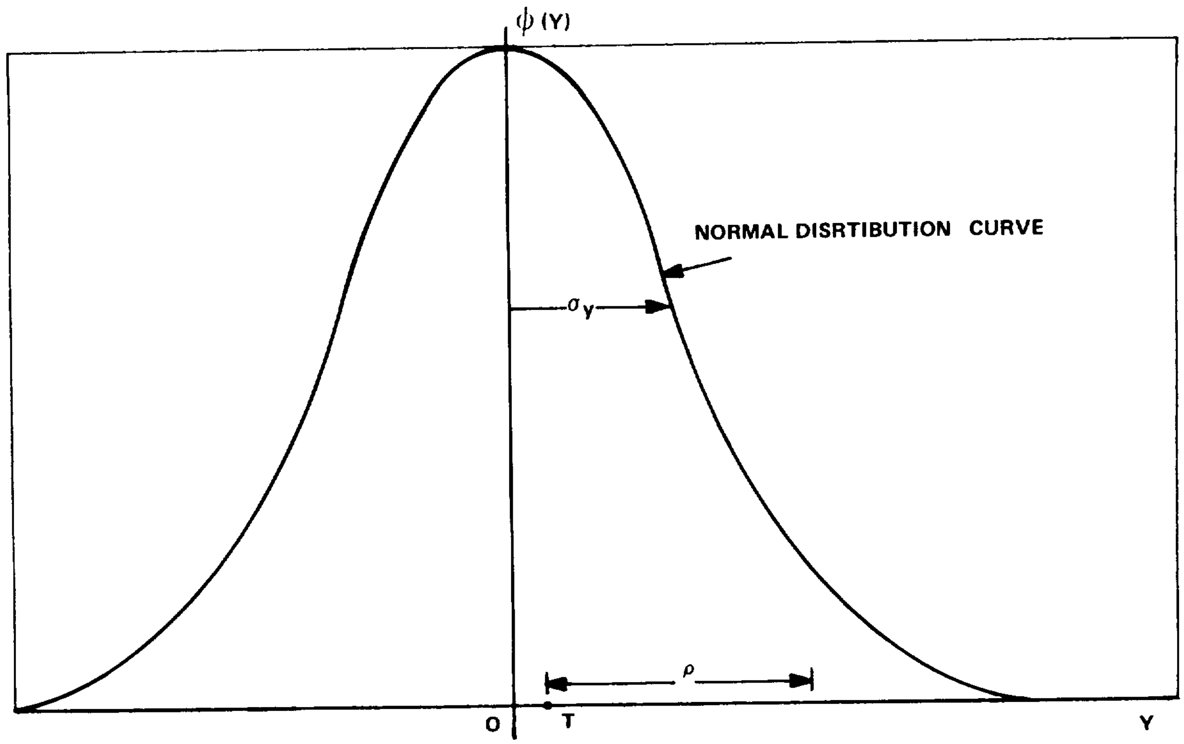

FIGURE 2A-

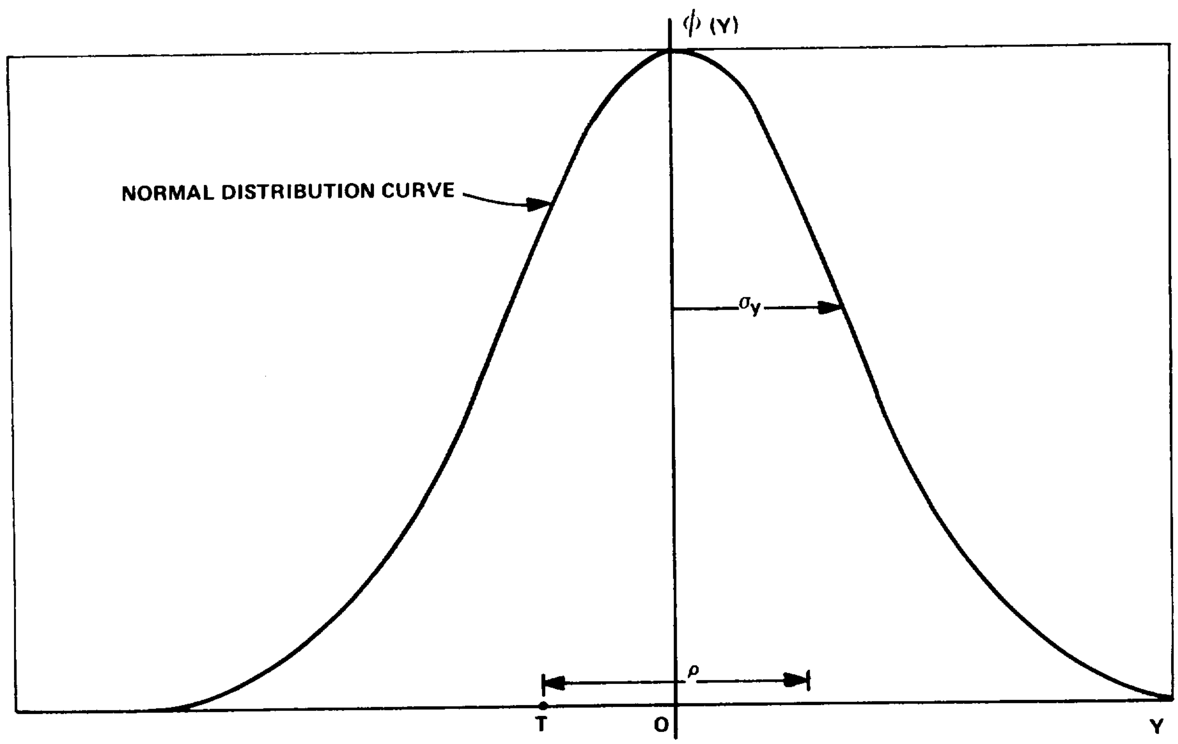

FIGURE 2B- ONE DIMENSIONAL REDESIGNATION TO POINT TARGET

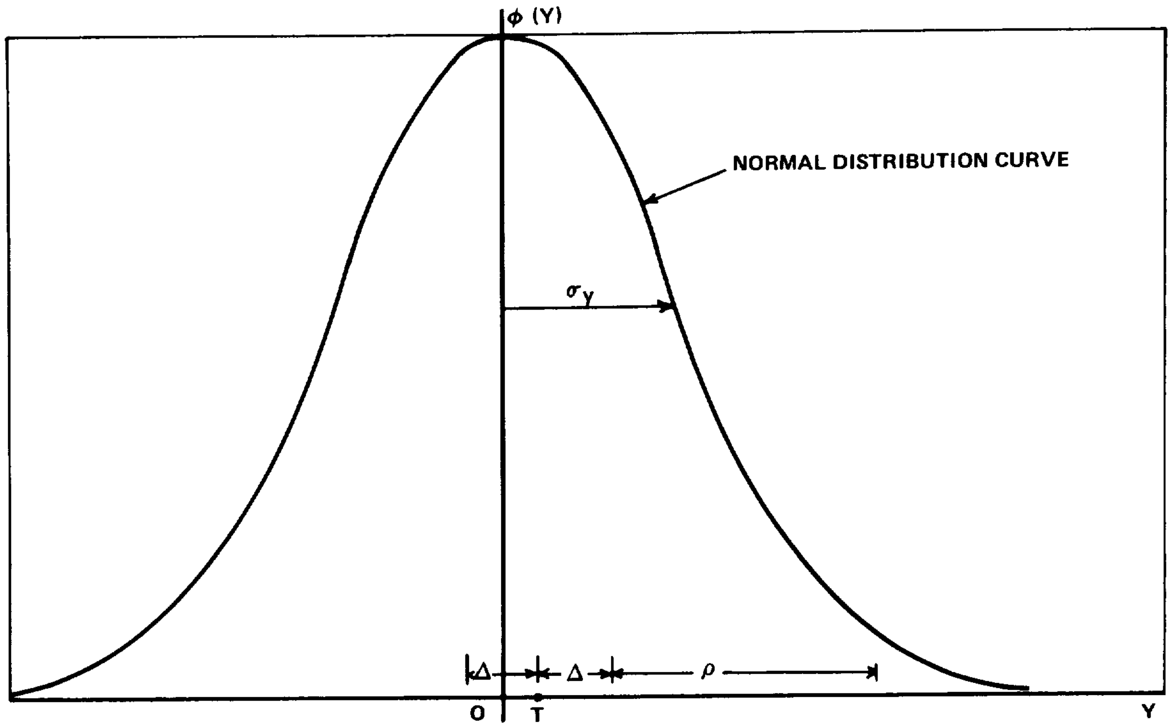

FIGURE 3A-

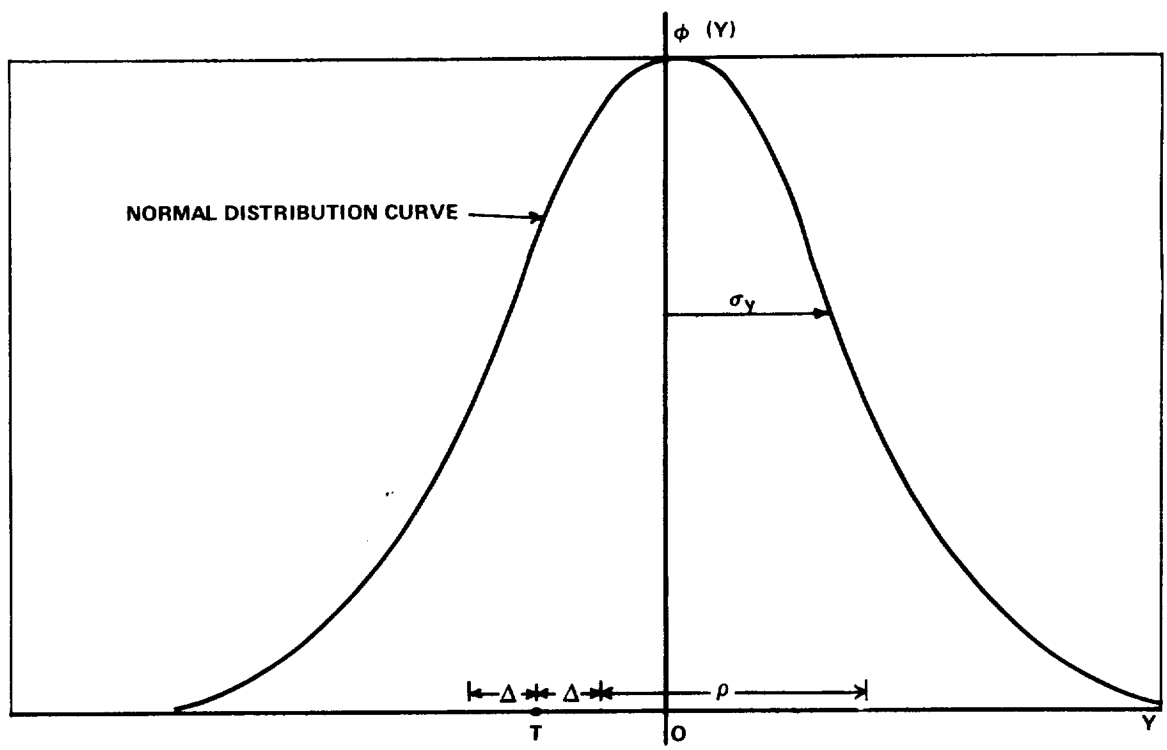

FIGURE 3B ONE DIMENSIONAL REDESIGNATION TO TARGET OF LENGTH 2Δ

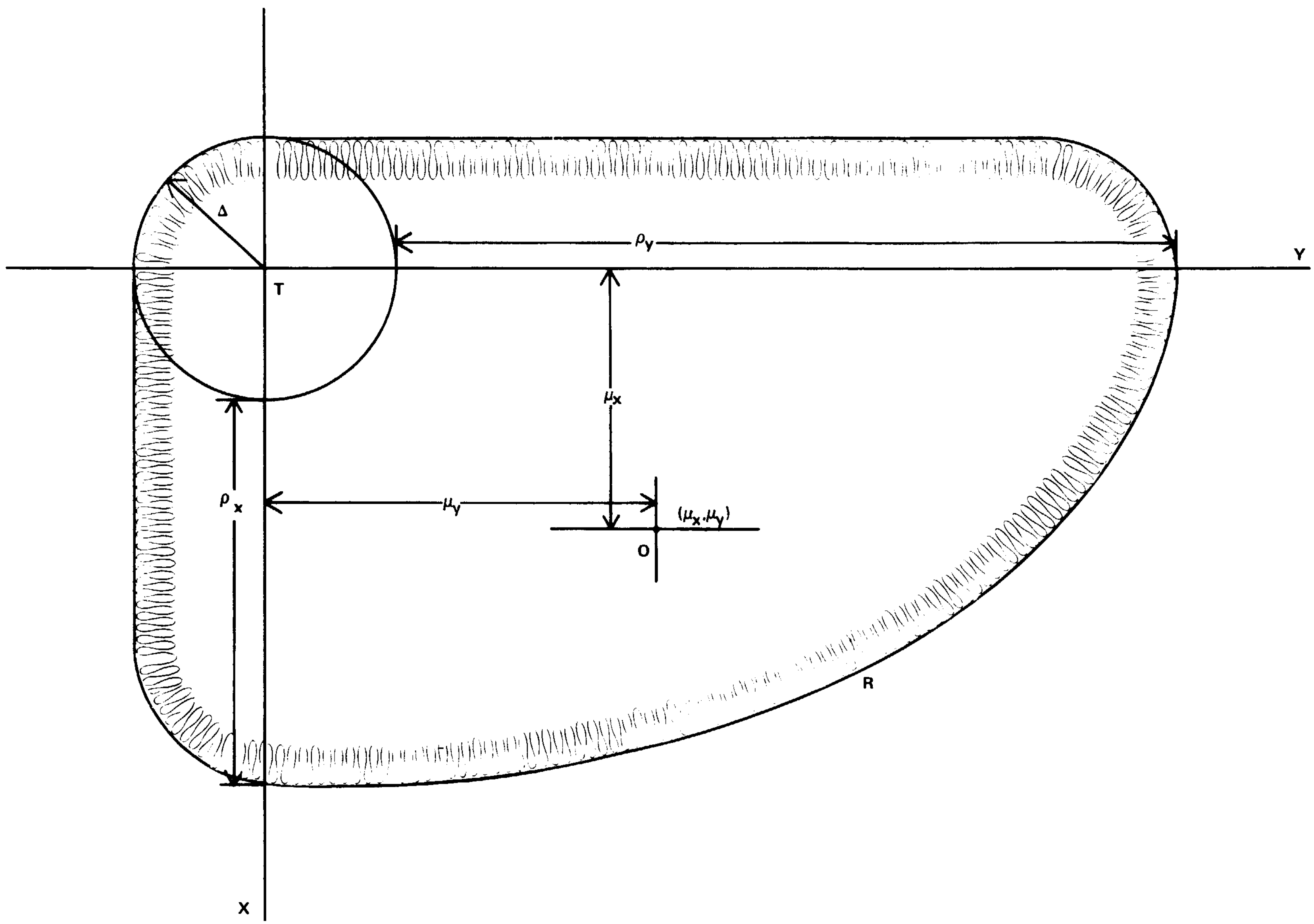

FIGURE 4 - TWO DIMENSIONAL REDESIGNATION TO TARGET OF RADIUS Δ

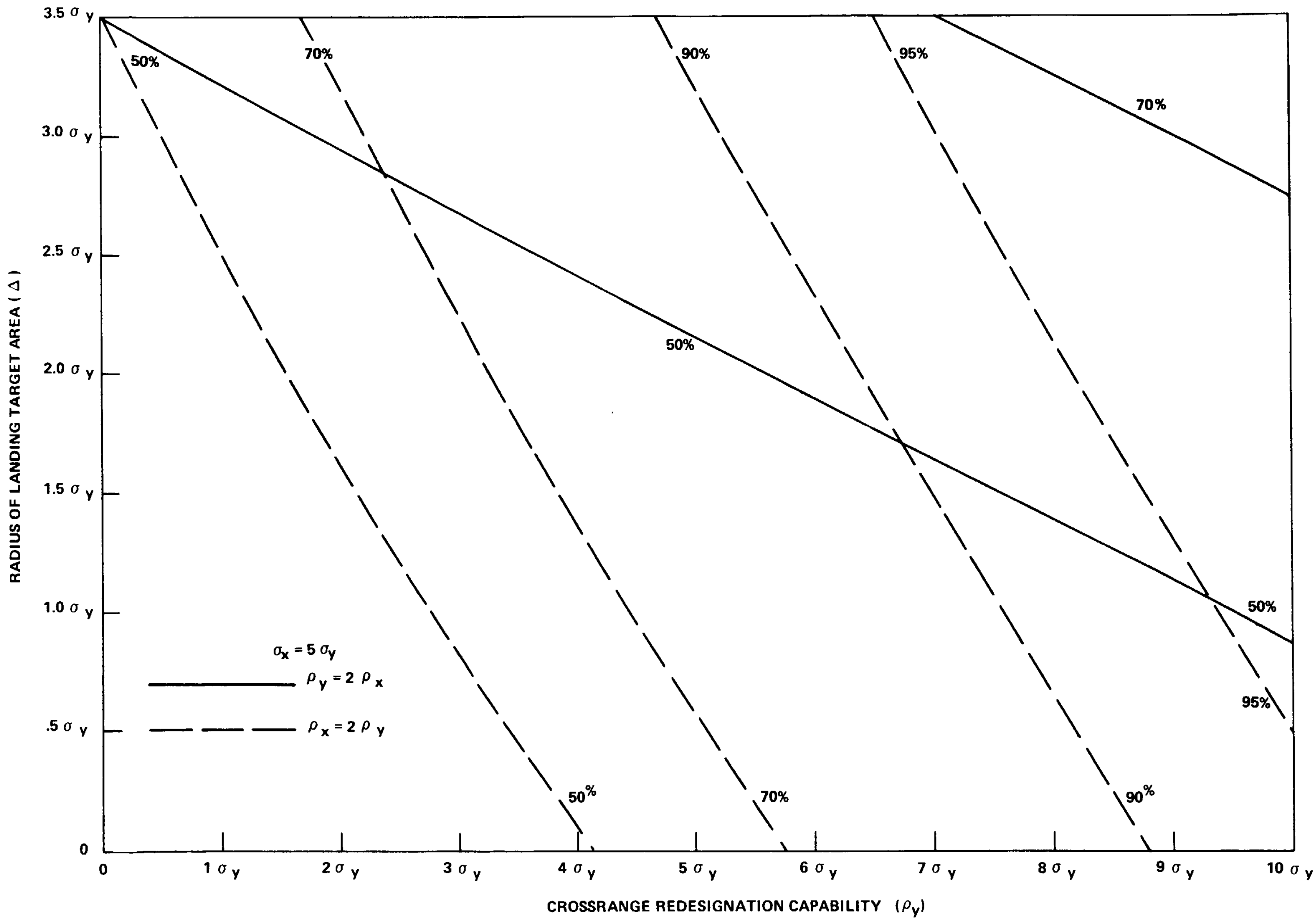

FIGURE 5A· PROBABILITY OF SUCCESS WITH ERROR ELLIPSE $\sigma_x = 5\,\sigma_y$

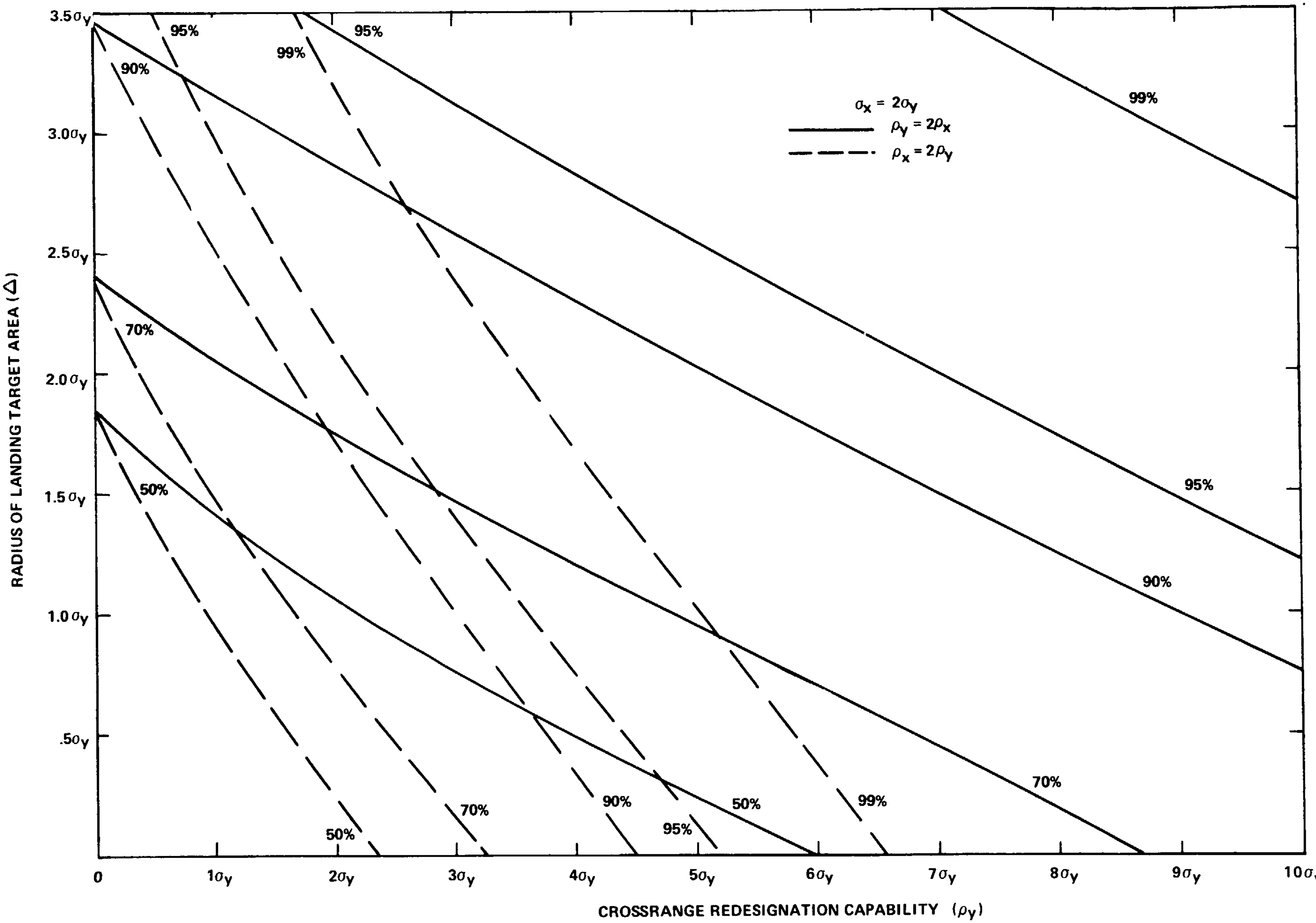

FIGURE 5B- PROBABILITY OF SUCCESS WITH ERROR ELLIPSE $\sigma_x = 2\sigma_y$

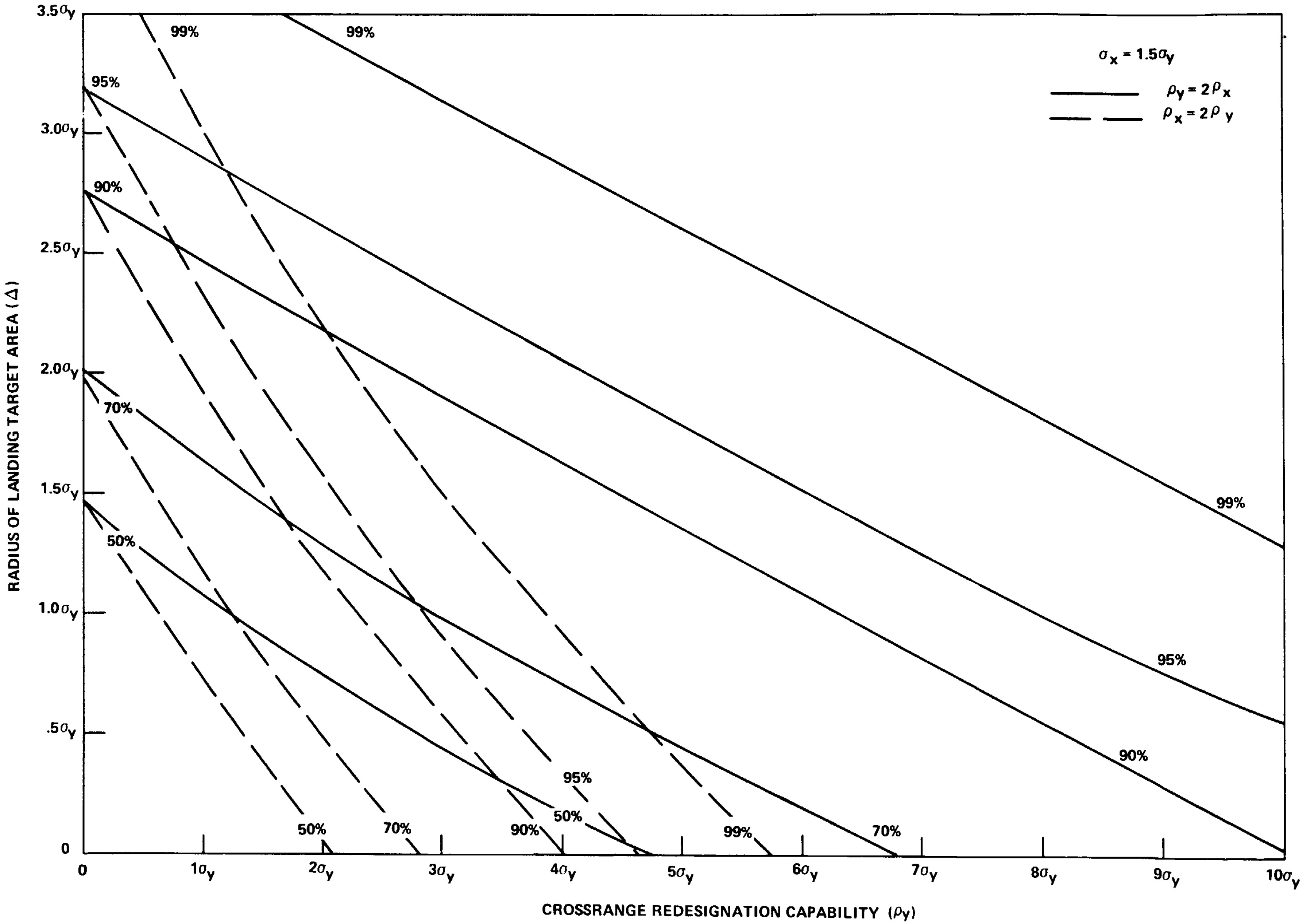

FIGURE 5C - PROBABILITY OF SUCCESS WITH ERROR ELLIPSE $\sigma_x = 1.5\ \sigma_y$

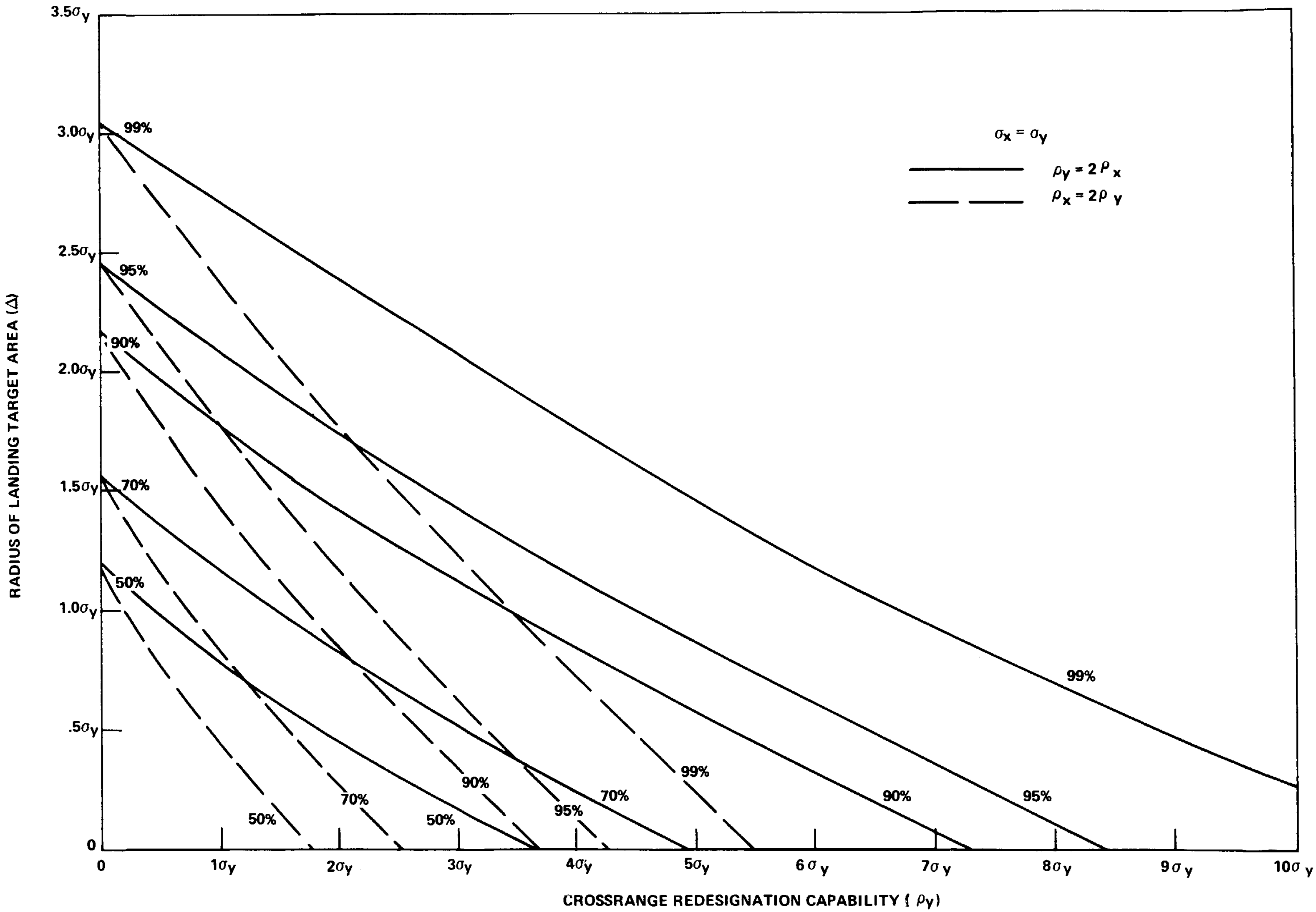

FIGURE 5D- PROBABILITY OF SUCCESS WITH ERROR ELLIPSE $\sigma_x =$ y

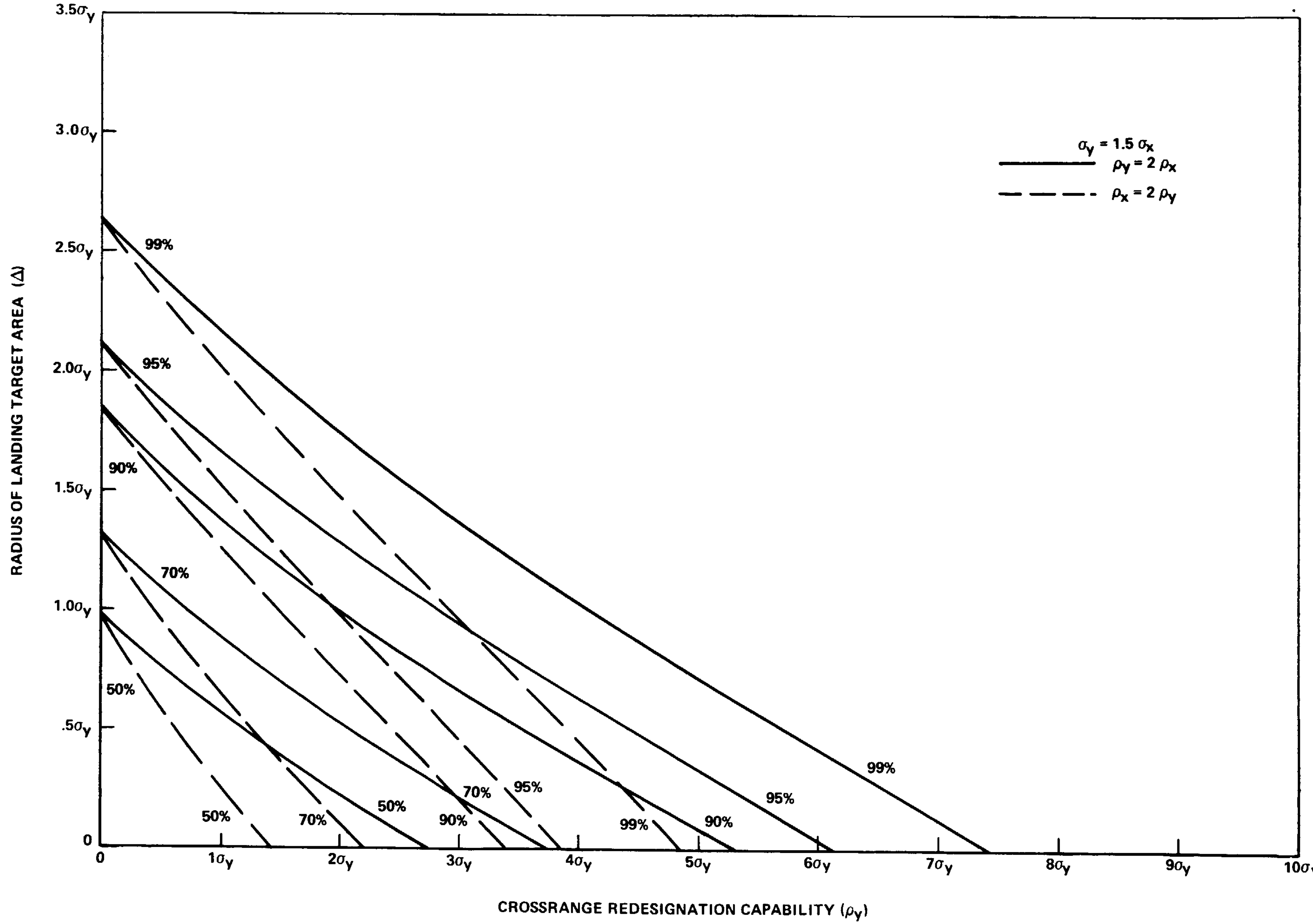

FIGURE 5E· PROBABILITY OF SUCCESS WITH ERROR ELLIPSE $\sigma_y = 1.5\,\sigma_x$

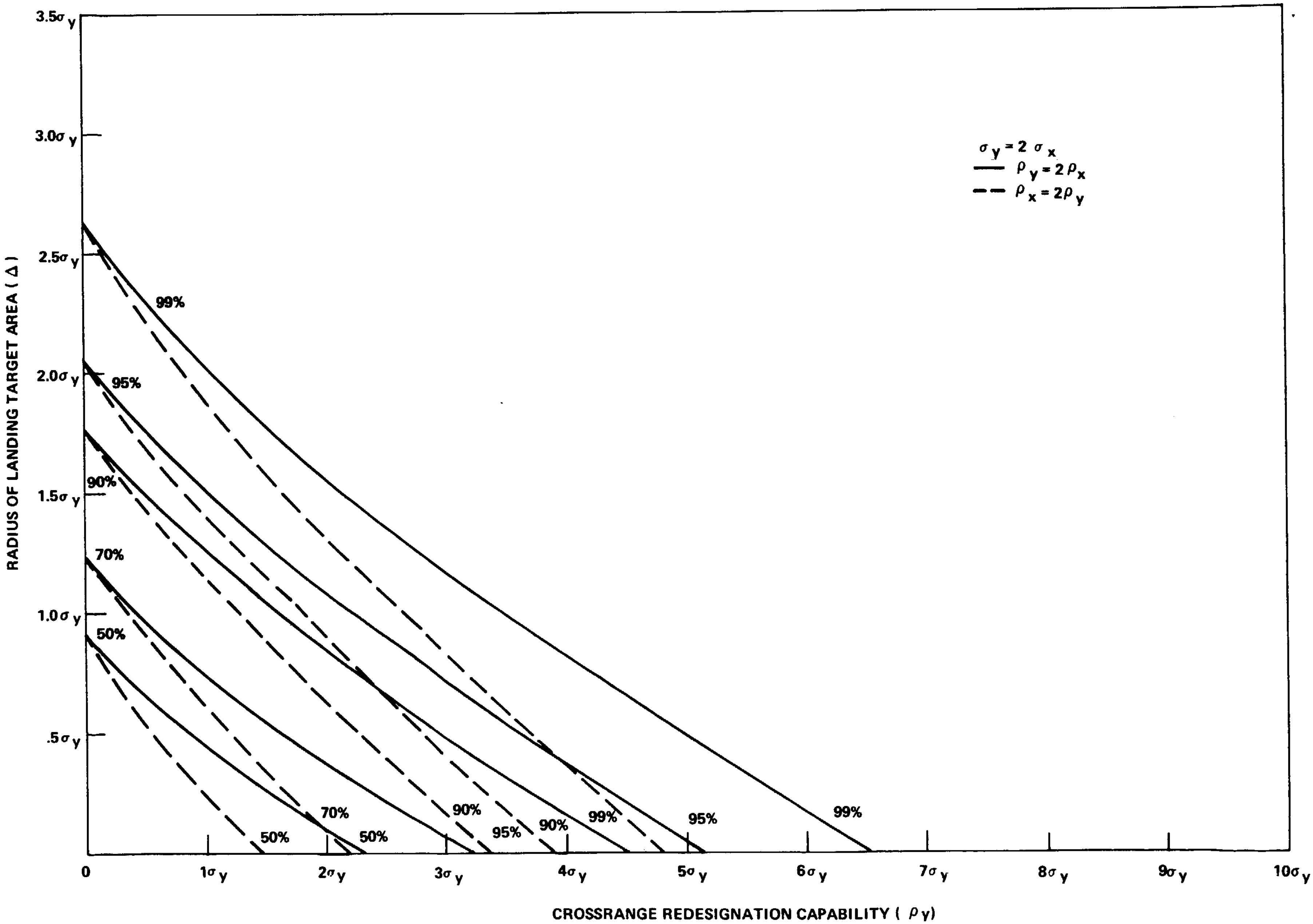

FIGURE 5f - PROBABILITY OF SUCCESS WITH ERROR ELLIPSE $\sigma_y = 2\sigma_x$

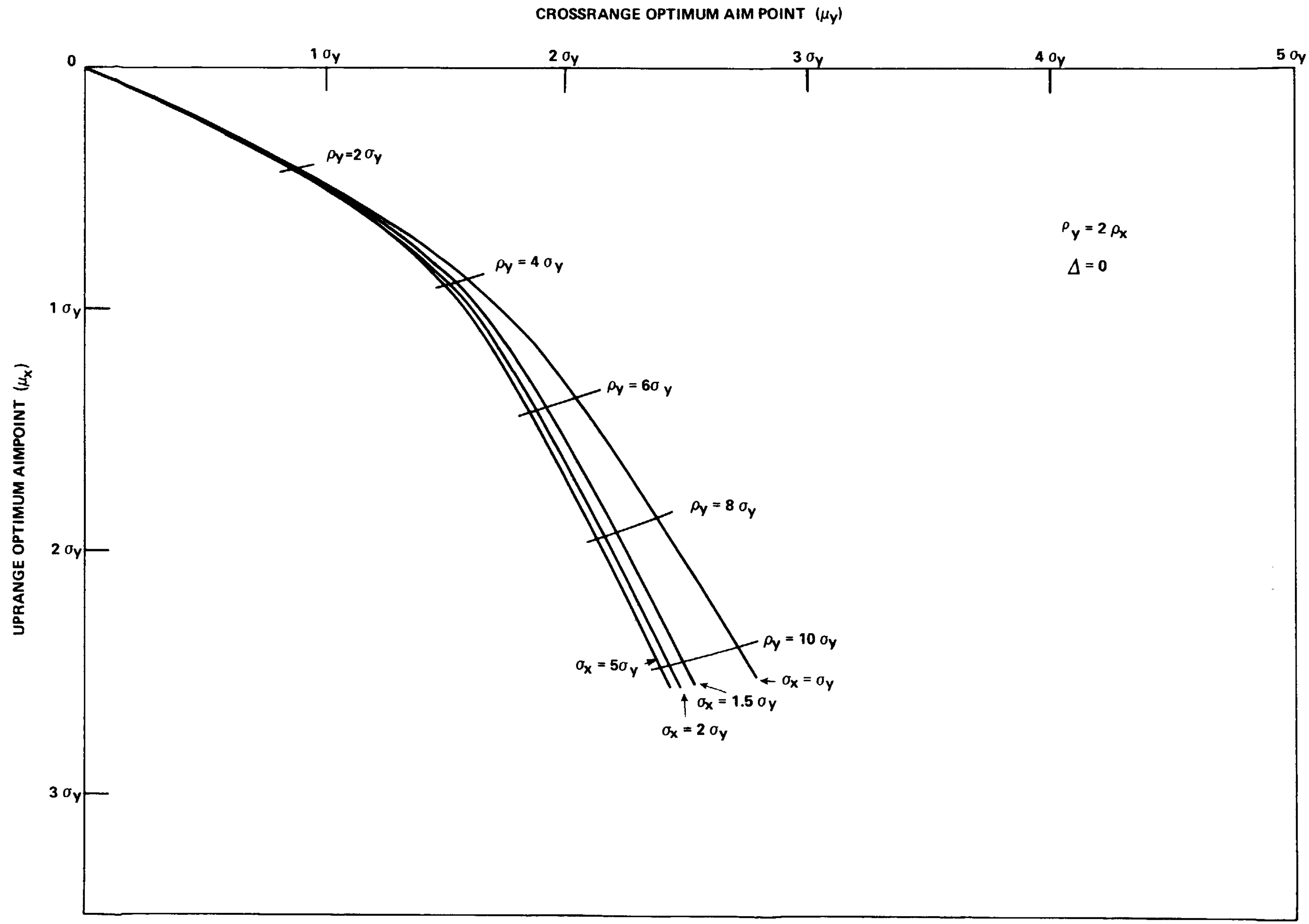

FIGURE 6a- OPTIMUM INITIAL AIM POINT LOCATION FOR POINT TARGET

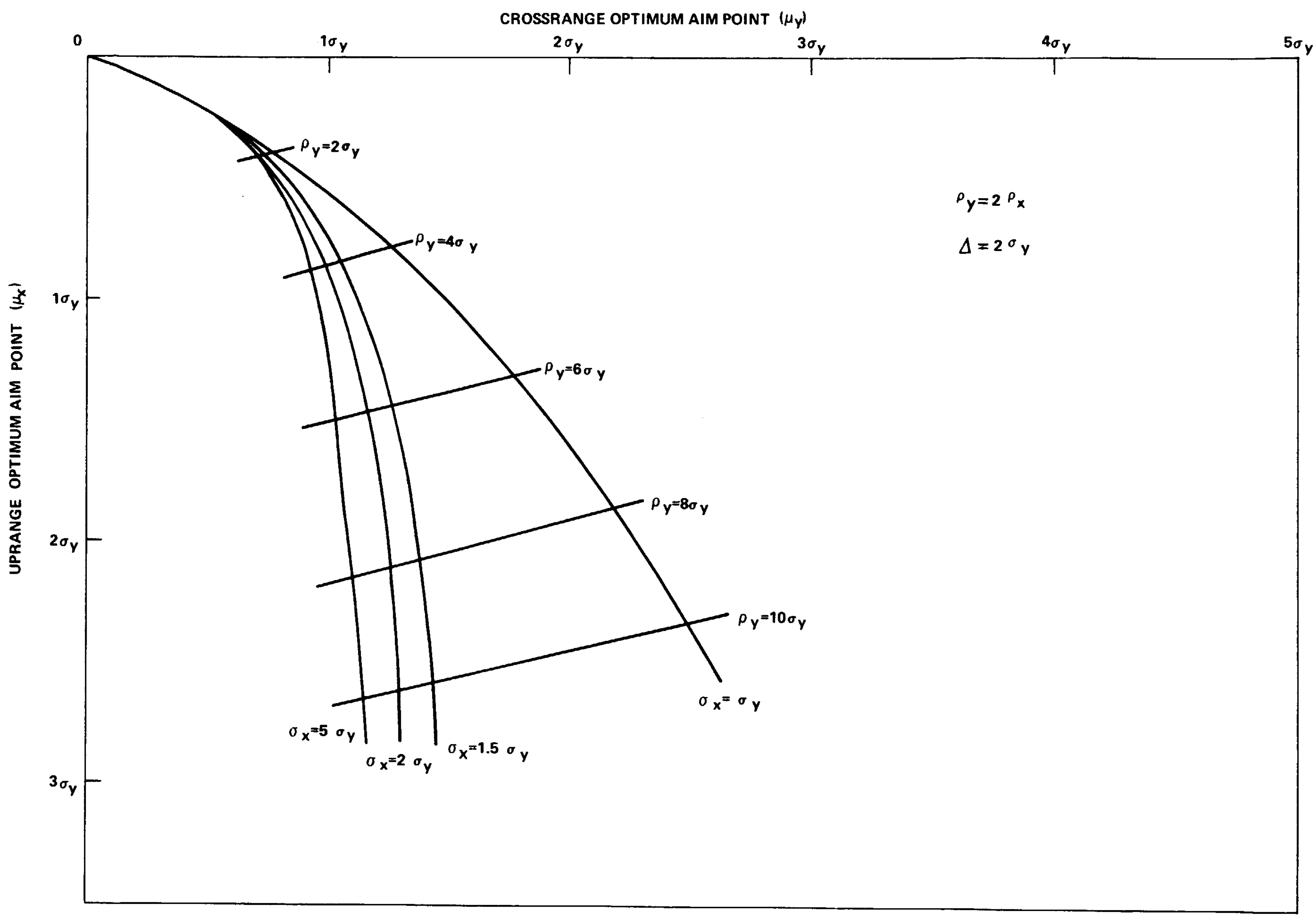

FIGURE 6b - OPTIMUM INITIAL AIM POINT LOCATION FOR TARGET OF RADIUS $2\sigma_y$

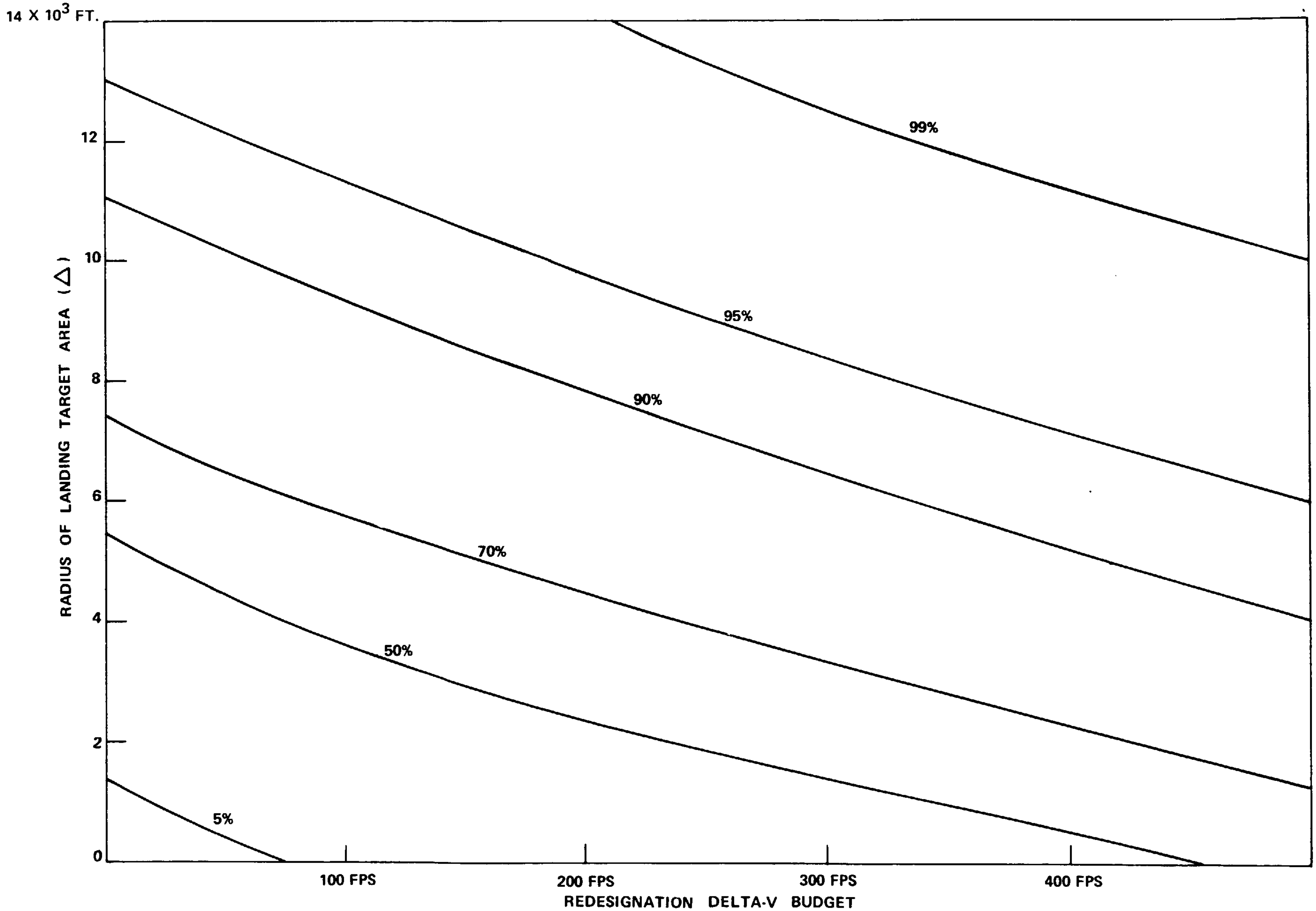

FIGURE 7 - REDESIGNATION AT 5000′ ALTITUDE FOR APOLLO 11 MISSION

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From: K. P. Klaasen

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